Measuring what matters in global value chain and value-added trade

Alessandro Borin  Michele Mancini

Bank of Italy

Singapore Management University
International fragmentation of production

Since ICT revolution and globalization: traditional trade statistics do not provide an adequate representation of supply and demand linkages among the economies anymore.
International fragmentation of production

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With the spread of GVCs, lots of new questions (e.g. measuring participation and positioning in global production sharing).

Need new macro data, new methods and new tools to answer old and new questions.
Contributions

**Methods** A better understanding of value-added and GVC trade:
- how to measure VA content at the aggregate, bilateral and bilateral-sectoral level, depending on the specific empirical issue;
- developing a measure of GVC-related trade;
- reconcile large part of the existing literature under one framework.
Motivation and contributions

Contributions

Methods A better understanding of value-added and GVC trade:

- how to measure VA content at the aggregate, bilateral and bilateral-sectoral level, depending on the specific empirical issue;
- developing a measure of GVC-related trade;
- reconcile large part of the existing literature under one framework.

Tools An easy way to compute measures:

- icio: Economic Analysis with Inter-Country Input-Output tables in Stata, `ssc install icio`;
- Dataset of country/sector measures based on WIOD, TiVA and EORA: World Bank WDR2020 website [Link](#)
New data
New data: from IO to ICIO

- **Input-Output tables**: production and consumption structure within an economy.
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- **In the last five years** Inter-Country Input-Output tables: generalization of IO tables, describe the sale and purchase relationships between producers and consumers *within and between economies*.
  In a national IO table, exports are sales to the “foreign sector”; in a ICIO table, exports are to country 1, county 2 . . .
Inter-Country Input Output table

A standard ICIO table

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
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<td>3</td>
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</tr>
</tbody>
</table>

Delivery (supply) of intermediates and final goods

Use of intermediate products and factors

Value added

Gross output

Motivation and contributions

New data

New methods

Borin, Mancini

Measuring GVC and value-added trade

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New data: several databases

World Input-Output Database (University of Groningen and WIIW)
- 43 countries + ROW
- 56 sectors
- 2000-2014 (previous release, 1995-2011)
- Based on official statistics
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**Trade in Value-Added** (OECD)
- 64 countries + ROW
- 36 sectors
- Incorporation of firm heterogeneity: (some) processing trade correction for China and Mexico.
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**Trade in Value-Added** *(OECD)*

- 64 countries + ROW
- 36 sectors
- Incorporation of firm heterogeneity: (some) processing trade correction for China and Mexico.

Several other databases available: EORA, EXIOBASE, ADB-MRIO, AIIOITs, GTAP.
New methods
ICIO model: general setup

This is the ICIO table with $G$ countries and $N$ sectors:

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<thead>
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<td></td>
<td>$Z_{i1}$</td>
<td>$Z_{i2}$</td>
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<tr>
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- $Z_{ij}$ is the $N \times N$ matrix of intermediate inputs produced in country $i$ and used in country $j$;
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<td>$Y_{22}$</td>
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<tr>
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<td>$Z_{G1}$</td>
<td>$Z_{G2}$</td>
<td>…</td>
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- $X_i$ is the $N \times 1$ vector of gross output produced in country $i$;
- $Y_{ij}$ is the $N \times 1$ vector of final goods and services completed in country $i$ and absorbed in country $j$;
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Value Added: $VA_1$ $VA_2$ ... $VA_G$

Total Output: $(X_1)'$ $(X_2)'$ ... $(X_G)'$
**ICIO model: general setup**

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<tr>
<td></td>
<td>( 1 )</td>
<td>( Z_{11} )</td>
<td>( Y_{11} )</td>
</tr>
<tr>
<td></td>
<td>( 2 )</td>
<td>( Z_{12} )</td>
<td>( Y_{12} )</td>
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<td>\ldots</td>
<td>\ldots</td>
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| Value Added | \( VA_1 \) | \( VA_2 \) | \ldots | \( VA_G \) |
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- $\mathbf{V}$ is the $1 \times GN$ vector of value added coefficients (i.e. $\mathbf{VA} \otimes (\mathbf{X}'))$;
- $\mathbf{E}_i$ is the $N \times 1$ vector of gross exports of country $i$. 
Trade in VA: seminal contributions

- Import content of exports (VS), Hummels et al. (2001).
  - Good measure of participation of a country in the downstream phases of international production chains.
  - Does not trace the origin of imports: some input comes from abroad, others are re-imported.
Trade in VA: seminal contributions

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- Adding information on the origin and the destination of gross flows, Koopman et al. (2008).

\[ u_N E_{s*} = V_s B_{ss} E_{s*} + \sum_{t \neq s} V_t B_{ts} E_{s*} \]

- Still gross accounting: include items that cross a country boundaries several times, i.e. double counted.
- Need to isolate double counting to measure net production (VA or GDP) embedded in exports.

Borin, Mancini
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\text{domestic content (DC}_{s*}) + \text{foreign content (FC}_{s*})
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Motivation and contributions

New data

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Tracing value-added in export flows

Separating ‘value-added’ from ‘double counted’
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Trade in VA: seminal contributions

Koopman et al. (2014) single out Domestic VA and Foreign VA
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Gross Exports ($E_{s*}$)

Domestic Content ($DC_{s*}$)

Foreign Content ($FC_{s*}$)

Caveat 1: KWW decomposition applies only to aggregate exports, limited scope for application.

Caveat 2: some of the components are imprecisely/incorrectly defined.

Caveat 3: the definition of DDC and that of FDC follow two different logics.
Trade in VA: seminal contributions

Koopman et al. (2014) single out Domestic VA and Foreign VA

Gross Exports ($E_{s*}$)

- Domestic Content ($DC_{s*}$)
  - Domestic Value Added ($DVA_{s*}$)
  - Domestic Double Counted ($DDC_{s*}$)
- Foreign Content ($FC_{s*}$)
  - Foreign Value Added ($FVA_{s*}$)
  - Foreign Double Counted ($FDC_{s*}$)

Notably, Koopman et al. (2014) combine the measures of the content of exports (i.e. (2)) with the index of 'Value Added Exports' proposed by Johnson and Noguera (2012). In particular, the latter is a subcomponent of the domestic value-added embedded in gross exports, the remaining part being the value-added that is finally absorbed by the exporting country itself (labeled 'Reflection' by Koopman et al., 2014).

Koopman et al. (2014) breakdown applies only to aggregate exports, while investigating on the value-added content of trade at the bilateral level (and/or sectoral level) is crucial in many cases. For instance, this method has limited scope of application in the assessment of trade policies and in many analyses of international production linkages. Moreover, some of the components of Koopman et al. (2014) breakdown are imprecisely defined, as extensively discussed in Section 5.2.

Our main contribution is to provide an exhaustive and rigorous value-added decomposition of exports at the aggregate, bilateral and sectoral level. To this aim,
Trade in VA: seminal contributions

Koopman et al. (2014) single out Domestic VA and Foreign VA

- Gross Exports ($E_{s*}$)
  - Domestic Content ($DC_{s*}$)
    - Domestic Value Added ($DVA_{s*}$)
    - Domestic Double Counted ($DDC_{s*}$)
  - Foreign Content ($FC_{s*}$)
    - Foreign Value Added ($FVA_{s*}$)
    - Foreign Double Counted ($FDC_{s*}$)
  - Value Added Exports ($VAX_{s*}$)
  - Reflection ($REF_{s*}$)

Caveat 1: KWW decomposition applies only to aggregate exports, limited scope for application.
Caveat 2: some of the components are imprecisely/incorrectly defined.
Caveat 3: the definition of DDC and that of FDC follow two different logics.
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Gross Exports \((E_{s*})\)

Domestic Content \((DC_{s*})\)

Foreign Content \((FC_{s*})\)

Domestic Value Added \((DVA_{s*})\)

Foreign Value Added \((FVA_{s*})\)

Domestic Double Counted \((DDC_{s*})\)

Foreign Double Counted \((FDC_{s*})\)

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Gross Exports \( (E_{s*}) \)

\[ \text{Domestic Content} \quad (\text{DC}_{s*}) \]

\[ \text{Domestic Value Added} \quad (\text{DVA}_{s*}) \]

\[ \text{Domestic Double Counted} \quad (\text{DDC}_{s*}) \]

\[ \text{Foreign Content} \quad (\text{FC}_{s*}) \]

\[ \text{Foreign Value Added} \quad (\text{FVA}_{s*}) \]

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  - \text{Domestic Value Added (} DVA_{s*} \text{)}
  - \text{Domestic Double Counted (} DDC_{s*} \text{)}
- \text{Foreign Content (} FC_{s*} \text{)}
  - \text{Foreign Value Added (} FVA_{s*} \text{)}
  - \text{Foreign Double Counted (} FDC_{s*} \text{)}

\[ \text{Value Added Exports (} VAX_{s*} \text{)} \]

\[ \text{Reflection (} REF_{s*} \text{)} \]

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Looking for a unique framework

Main problem: there is not a unique correct method to separate VA from double counting, especially in disaggregated trade flows (Wang et al. 2013; Los et al. 2016; Johnson 2018).
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E. g.: KWW decomposition follow a country-level perspective for DVA and world-level perspective for FVA.
Different methods for different questions

Why several perspectives? It really depends to the economic question to address:
Different methods for different questions

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⇒ What part of a country’s GDP is absorbed abroad?
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⇒ What part of a country’s exports/output/value-added/final goods production is GVC? Country-level (Borin and Mancini, 2015, 2016; Wang et al., 2018)

⇒ What part of a country’s GDP is affected by an exogenous shock on imports of a partner? Bilateral-level (Johnson 2018; Borin and Mancini, 2019)

⇒ What part of a country’s GDP is affected by a tariff imposed by an importer in a given sector? Sectoral/bilateral level (Borin and Mancini, 2019)
Total exports, country level perspective

Starting from the exports decomposition in terms of contents:

\[ u_N E_{s*} = V_s B_{ss} E_{s*} + \sum_{t \neq s} V_t B_{ts} E_{s*}, \]

we single out VA, tracing the very first time it is exported by country s.
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we single out VA, tracing the very first time it is exported by country \( s \).

How? Modify \( B = I + A + A^2 + A^3 + \ldots + A^n \) \( n \to \infty \).

\[
A^s = \begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1s} & \cdots & A_{1G} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & A_{ss} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
A_{G1} & A_{G2} & \cdots & A_{Gs} & \cdots & A_{GG}
\end{bmatrix}
\]

\[ B^s = (I - A^s)^{-1}. \]

Note that \( B_{ss}^s = (I - A_{ss})^{-1} \).
Total exports, country level perspective

Now we can separate DVA from double counting:

\[ u_N E_{s*} = V_s B_{ss}^s E_{s*} + V_s B_{ss}^s \sum_{j \neq s} A_{sj} B_{js} E_{s*} + \sum_{t \neq s} V_t B_{ts}^s E_{s*} + \sum_{t \neq s} V_t B_{ts}^s \sum_{j \neq s} A_{sj} B_{js} E_{s*} \]
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Then, it is possible to go further and identify the country of absorption.
Total exports, country level perspective

Main features:

- DVA is standard, in line with Koopman et al. (2014), Los et al. (2016), Wang et al. (2013).
Total exports, country level perspective

Main features:

- DVA is standard, in line with Koopman et al. (2014), Los et al. (2016), Wang et al. (2013).
- FVA is different from Koopman et al. (2014) and Wang et al. (2013).
Total exports, country level perspective

An example on FVA with a world level perspective (KWW, WWZ). Looking at country A’s exports:
Total exports, country level perspective

An example on FVA with a world level perspective (KWW, WWZ). Looking at country A’s exports:

![Diagram showing the flow of exports from Z to Y, passing through A and B.]

World perspective is unsatisfactory if the goal is to trace foreign GDP in exports.
Total exports, country level perspective

An example on FVA with a world level perspective (KWW, WWZ). Looking at country A’s exports:

World perspective is unsatisfactory if the goal is to trace foreign GDP in exports.
Bilateral exports, country level perspective

Easy to move from the aggregate to the bilateral level:
Bilateral exports, country level perspective

Easy to move from the aggregate to the bilateral level:

\[
\begin{align*}
\text{domestic content (DC}_{sr}\text{)} & = \text{domestic value added (DVA}_{sr}\text{)} + \text{domestic double counted (DDC}_{sr}\text{)} \\
\text{foreign content (FC}_{sr}\text{)} & = \text{foreign value added (FVA}_{sr}\text{)} + \text{foreign double counted (FDC}_{sr}\text{)}
\end{align*}
\]

- The items computed in the decompositions of bilateral trade flows in a country-level perspective sum up to the decomposition of aggregate exports.
- Through this approach we can single put the value of production that crosses just one border.
Bilateral decomposition: a scheme

Exporter (and origin of domestic VA)  →  Direct Importer

BILATERAL FLOW
Bilateral decomposition: a scheme
Bilateral decomposition: a scheme
Bilateral decomposition: a scheme

- **Country of origin of foreign VA**
- **Exporter (and origin of domestic VA)**
- **Direct Importer**
- **Destination of re-export**
- **Country of completion of final products**

Other possible stages of production
Bilateral decomposition: a scheme

- Country of origin of foreign VA
- Exporter (and origin of domestic VA)
- Destination of final products
- Destination market
- Direct Importer
- Destination of re-export

Other possible stages of production
Bilateral decomposition: a scheme

- Country of origin of foreign VA
- Exporter (and origin of domestic VA)
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Bilateral decomposition: a scheme

Country of origin of foreign VA

Exporter (and origin of domestic VA)

Direct Importer

Country of completion of final products

Destination of re-export

Destination market
Bilateral exports, country level perspective

Main features:

- DVA absorbed directly by the importer:
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- DVA absorbed directly by the importer:

\[
\text{DAVAX}_{sr} = V_s (I - A_{ss})^{-1} Y_{sr} + V_s (I - A_{ss})^{-1} A_{sr} (I - A_{rr})^{-1} Y_{rr}
\]
Bilateral exports, country level perspective

Main features:

- DVA absorbed directly by the importer:

\[
\text{DAVA}_s = \mathbf{V}_s (\mathbf{I} - \mathbf{A}_s)^{-1} \mathbf{Y}_s + \mathbf{V}_s (\mathbf{I} - \mathbf{A}_s)^{-1} \mathbf{A}_s (\mathbf{I} - \mathbf{A}_r)^{-1} \mathbf{Y}_r
\]

Going through bilateral partners is the only way to capture the portion of a country’s exports that is re-exported.
Bilateral exports, country level perspective

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Going through bilateral partners is the only way to capture the portion of a country’s exports that is re-exported.

- GVC-related trade defined as the value of production that cross more than one border:

\[
\text{GVC}_s = \sum_{r \neq s}^{G} u_N E_{sr} - \text{DAVAX}_{sr}
\]
Bilateral exports, country level perspective

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- DVA absorbed directly by the importer:

\[ \text{DAVAX}_{sr} = V_s(I - A_{ss})^{-1}Y_{sr} + V_s(I - A_{ss})^{-1}A_{sr}(I - A_{rr})^{-1}Y_{rr} \]

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- GVC-related trade defined as the value of production that cross more than one border:

\[ \text{GVC}_s = \sum_{r \neq s} G_u N E_{sr} - \text{DAVAX}_{sr} \]

Koopman et al. (2014) and Wang et al. (2013) are not able to trace exporter-importer linkages; cannot separate the DVA absorbed in the first destination or re-exported to other countries.
DVA absorbed directly by the importer

The goal is to find DVA re-exported, i.e. the difference between DVA and DVA absorbed directly by the importer. Focus on the latter:

\[
\text{DAVAX}_s = \sum_{r \neq s} G V_s (I - A_{ss})^{-1} Y_{sr} + \sum_{r \neq s} G V_s (I - A_{ss})^{-1} A_{sr} (I - A_{rr})^{-1} Y_{rr}
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- Koopman et al. (2014):

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- Wang et al. (2017, 2018): acknowledge that our approach is the most appropriate to single out “traditional trade”.
## A comparison with KWW

WIOD 2014 data.

<table>
<thead>
<tr>
<th>DEU exports</th>
<th>(1) KWW</th>
<th>(2) BM</th>
<th>(3) diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAVAX</td>
<td>61.9</td>
<td>53.7</td>
<td>8.2</td>
</tr>
<tr>
<td>DVA re-exported</td>
<td>9.9</td>
<td>18.1</td>
<td>-8.2</td>
</tr>
<tr>
<td>Domestic DC</td>
<td>1.4</td>
<td>1.4</td>
<td>0.0</td>
</tr>
<tr>
<td>FVA</td>
<td>19.2</td>
<td>26.1</td>
<td>-6.9</td>
</tr>
<tr>
<td>Foreign DC</td>
<td>7.5</td>
<td>0.6</td>
<td>6.9</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>100.0</td>
<td>100.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Consider the exports of country $s$, $E_{s*}$. Single out GVC backward and forward participation:

$$GVCB_s = \sum_{r \neq s} DD_{sr} + \sum_{t \neq r} FC_{sr}$$

Remark 1: $GVCB_s = VS_s$ (Hummels et al. 2001; WWZ 2018; KWW 2014 exclude DDC)

Remark 2: $GVCF_s$ differs from $VS_1$ as defined by KWW 2014: $G_{r \neq s} V_B s r E_r$ not a portion of country $s$ exports (traced in $E_{s*}$). Increases with the upstreamness of a country: if $s$ goods are re-exported by other countries many times, country $s$ index increases.
Consider the exports of country $s$, $E_{s^*}$. Single out GVC backward and forward participation:

$$GVC_B_s = \sum_{r\neq s}^G DDC_{sr} + \sum_{t\neq r}^G FC_{sr}$$

$$GVC_F_s = \sum_{r\neq s}^G DVA_{sr} - \sum_{r\neq s}^G DAVAX_{sr}$$
GVC participation: backward and forward

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GVC participation: backward and forward

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\sum_{r \neq s} V_s B_{sr} E_{r*}
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GVC participation: backward and forward

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- Not a portion of country $s$ exports (traced in $E_{r*}$)
- Increases with the upstreamness of a country: if $s$ goods are re-exported by other countries many times, country $s$ index increases.
Motivation and contributions

GVC participation: evidence

GVC trade
(share on total trade)

Low income
Middle income
High income

Bilateral exports, bilateral level perspective

What part of a country’s value-added is involved in a certain bilateral trade relation?

$$B_{sr} = A_{sr} \cdot \cdots \cdot A_{sr}$$

$$B_{sr} = (I - A_{sr})^{-1}.$$
Bilateral exports, bilateral level perspective

What part of a country’s value-added is involved in a certain bilateral trade relation?

We modify $B$ to single out value-added and double counted terms only within a certain bilateral relation:
Motivation and contributions

New data

New methods

Bilateral exports, bilateral level perspective

What part of a country’s value-added is involved in a certain bilateral trade relation?

We modify $\mathbf{B}$ to single out value-added and double counted terms only within a certain bilateral relation:

$$\mathbf{A}^{sr} = \begin{bmatrix} \mathbf{A}_{11} & \cdots & \mathbf{A}_{1s} & \cdots & \mathbf{A}_{1r} & \cdots & \mathbf{A}_{1G} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{A}_{s1} & \cdots & \mathbf{A}_{ss} & \cdots & \mathbf{0} & \cdots & \mathbf{A}_{sG} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{A}_{G1} & \cdots & \mathbf{A}_{Gs} & \cdots & \mathbf{A}_{Gr} & \cdots & \mathbf{A}_{GG} \end{bmatrix}.$$
Bilateral exports, bilateral level perspective

What part of a country’s value-added is involved in a certain bilateral trade relation?

We modify $\mathbf{B}$ to single out value-added and double counted terms only within a certain bilateral relation:

$$\mathbf{A}^{sr} = \begin{bmatrix}
  \mathbf{A}_{11} & \cdots & \mathbf{A}_{1s} & \cdots & \mathbf{A}_{1r} & \cdots & \mathbf{A}_{1G} \\
  \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
  \mathbf{A}_{s1} & \cdots & \mathbf{A}_{ss} & \cdots & \mathbf{0} & \cdots & \mathbf{A}_{sG} \\
  \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
  \mathbf{A}_{G1} & \cdots & \mathbf{A}_{Gs} & \cdots & \mathbf{A}_{Gr} & \cdots & \mathbf{A}_{GG}
\end{bmatrix}.$$ 

$$\mathbf{B}^{sr} = (\mathbf{I} - \mathbf{A}^{sr})^{-1}.$$
Bilateral exports, bilateral level perspective

What part of a country’s value-added is involved in a certain bilateral trade relation?

We can separate VA from double counting as follows:

\[
\mathbf{u}_N \mathbf{E}_{sr} = \mathbf{V}_s \mathbf{B}_{ss}^fr \mathbf{E}_{sr} + \mathbf{V}_s \mathbf{B}_{ss}^fr \mathbf{A}_{sr} \mathbf{B}_{rs} \mathbf{E}_{sr} + \sum_{t \neq s} \mathbf{V}_s \mathbf{B}_{ts}^fr \mathbf{E}_{sr} + \sum_{t \neq s} \mathbf{V}_t \mathbf{B}_{ts}^fr \mathbf{A}_{sr} \mathbf{B}_{rs} \mathbf{E}_{sr}. 
\]

- **domestic content (DC)\(_{sr}\)**
- **foreign content (FC)\(_{sr}\)**
- **bilateral perspective**
  - **DVA\(_{sr}\)**
  - **DDC\(_{sr}\)**
  - **FVA\(_{sr}\)**
  - **FDC\(_{sr}\)**
What part of a country’s value-added is involved in a certain bilateral trade relation?

We can separate VA from double counting as follows:

\[ u_N E_{sr} = V_s B_{ss}^s E_{sr} + V_s B_{ss}^s A_{sr} B_{rs} E_{sr} + \sum_{t \neq s} V_t B_{ts}^s E_{sr} + \sum_{t \neq s} V_t B_{ts}^s A_{sr} B_{rs} E_{sr} \]

It is also possible identify the country/sector of origin/absorption.
Sectoral-bilateral perspective

What is the value-added that enters in the exports from a country to another in a given sector? and where did it originated?
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We modify $B$ to single out ‘value-added’ and ‘double counted’ terms only within a certain sectoral-bilateral relation:

$$B_{sr,n} = (I - A_{sr,n})^{-1}$$
What is the value-added that enters in the exports from a country to another in a given sector? and where did it originated?

We can separate VA from double counting as follows:

\[
e_{sr,n} = V_s B_{ss}^{sr} E_{sr,n} + V_s B_{ss}^{sr} A_{sr} B_{rs} E_{sr,n} + \sum_{t \neq s} V_t B_{ts}^{sr} E_{sr,n} + \sum_{t \neq s} V_t B_{ts}^{sr} A_{sr} B_{rs} E_{sr,n}
\]

sectoral-bilateral perspective

\[
DVA_{sr} + \text{domestic content (DC}_{sr})
\]

\[
DDC_{sr} + \text{foreign content (FC}_{sr})
\]

sectoral-bilateral perspective

\[
FVA_{sr} + \text{bilateral perspective}
\]

\[
FDC_{sr}
\]
Sectoral-bilateral perspective

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It is also possible identify the country/sector of origin/absorption.